

# THE TWO-PITCH CHINESE BELL DURING THE SHANG- AND CHOU DYNASTY

Dr. André Lehr  
National Carillon Museum  
Ostaderstraat 23, NL-5721 WC Asten  
E-mail: alehr@iae.nl

## Aim of the Study

In the China of before our era, the bell did not have a rotational symmetric cross section, but a cross section that can be better described, albeit not precisely, as two segments of a circle placed against one another. As a result of this the partial tones split into a low and high component. The hum note and the overtones are thus doubled. The interval between both hum notes amounts to a third. Although one could assume that this interval is determined by the eccentricity of the cross section, it appears in practice to only play a minor role. For that reason, a study was conducted in order to establish which parameters play the most important part in determining the interval.

## Introduction

During the Shang (c.1520-c.1030 BC) - and Chou (c.1030-221 BC) - dynasty China had a thriving bell culture<sup>1</sup>. Bells up to heights of even 1½ m were used for all kinds of purposes; although as the rule the bells were generally smaller than 50 cm. The bells' most important task was undoubtedly musical performances. Rows of bells (Figure 1) were played often with other instruments that frequently included the flute and in particular the drum. This is also reflected in the literature of that period. Numerous references are to be found in the *Book of Odes*, the *I Ching*, a collection of poems that dates back to the second millennium before Christ. Thus we read for example<sup>2</sup>:

*The bells and drums sound in harmony,  
The sounding stones and flutes blend their notes,  
Abundant blessing is sent down.*

And elsewhere:

*I have here an admirable guest,  
And with all my heart I love him.  
The bells and drums have been arranged in order,  
And all the morning will I pledge him.*

These performances took place in palaces and temples, therefore indoors. That is why these bells were not struck with a heavy iron clapper, as later became customary in Europe, but with a light wooden hammer. Figure 2 shows such a hammer as was found in the tomb of the

---

<sup>1</sup> For this subject see: André Lehr, *Klokken en klokkenspel in het oude China tijdens de Shang- en Chou-dynastie* (Althanasius Kircher-Stichting 1985). Lothar von Falkenhausen, *Suspended Music. Chime-Bells in the Culture of Bronze Age China* (Berkeley, Los Angeles, Oxford, 1993).

<sup>2</sup> James Legge, *The Chinese Classics*. Vol.IV-Part I, *The She-King* (London, 1871), p.279 and 579.

Marquis Yi (433 BC or shortly afterwards). The sixty-five bells that were found in this uncommonly rich grave will be examined in more detail hereafter.

The way in which one played on these carillons has also been preserved (Figures 3-4). On both pictures that have been copied from bronze vases from the Chou dynasty, the way of playing is clearly shown<sup>3</sup>. In the lowest edge one sees the musicians with hammers play both a row of bells and a lithophone. As a result of this only the hum note was of musical importance. Overtones were not sufficiently audible in order to actually pay attention to them. Any references hereinafter to the tone of a Chinese bell will always mean the hum note. The bells from this period have a noticeable characteristic (Figure 5). Their cross section is not circular but roughly comprises two segments of a circle placed against one another. For the purpose of this article a segment of a circle will mean a part of a circle that is enclosed by an arc and a chord. Therefore, a bell with such a cross section has a long and short axis. This sometimes provokes the comment that this cross section is an ellipse. However this is not necessarily the case as the sides of a bell are not round but always pointed. Hence it is also referred to as an almond-shaped cross section. It will become clear further on in this article why it cannot be an ellipse.

As the old Chinese bell is not rotational symmetric, this results in a splitting up of all partial tones in the same way as can be heard for example in a tea cup. If one taps the handle then one will hear a lower tone than if one strikes the cup 45° further in the circumference. A similar occurrence can also be established in European bells, albeit on a much more modest scale. If a bell has an asymmetry somewhere in the circumference, for example as a result of a blow hole, the partial tones will also split. But in that case the difference in frequency is only slight, which is why these two tones are heard as a warble. This phenomenon can be heard in numerous bells, particularly in the hum note.

In a bell with a warble the interval between both hum notes is only slight, generally less than ten cents. However, it should be clear that when the deviation of the rotational symmetry of the bell is gradually increased that difference also increases. If that process is gradually, beyond a whole tone for example, then no more warbles can be heard, but both tones can be heard separately. And that is the case with the antique Chinese bell, thanks to the considerable deviation from the circular cross section. Figure 6 shows the accompanying horizontal modes of vibration of both hum notes. The drawn line is the bell when stationary; the dashed line is the one extreme position during the vibration; the dash-point line is the other extreme position. One can see therefore that each point of the bell wall swings back and forth between both extreme positions.

There are only four points where the bell wall is stationary for the fundamental tone. These points are called nodes, whereas the points with the greatest deviation are called antinodes. The sketch on the left-hand side shows the lowest fundamental tone that was called *sui*. This tone has its antinodes in the middle of the waist and at the sides of the bell. The other, higher hum note is called *ku*. This tone has its antinodes approximately halfway between the front of the bell and the pointed side. Or to be exact, the *ku* has its antinodes where the *sui* has its nodes and vice versa. This means therefore that when one strikes the bell precisely in an antinode of the *sui*, the *ku* is not heard, and again vice versa. And finally, if one strikes the bell halfway in an antinode of the *sui* and an antinode of the *ku*, both tones will be activated. Thanks to the fact that these tones lie approximately one third from one another, a person with an analytical ear can easily listen to them separately from one another.

---

<sup>3</sup> Lothar von Falkenhausen, *Suspended Music. Chime-Bells in the Culture of Bronze Age China* (Berkeley, Los Angeles, Oxford, 1993), p.30, 212.

### The sui/ku interval versus the eccentricity of the bell

Both fundamental tones usually form a third. It would therefore seem an obvious conclusion that the size of this interval is determined by the eccentricity of the bell. In order to examine this idea more closely, sixty-five bells were chosen that were found in the previously mentioned tomb of the Marquis Yi<sup>4</sup>. This grave with a remarkable amount of objects, including numerous musical instruments, dates from 433 BC or shortly afterwards. The tomb is located in the vicinity of Suihsien city in the present Hubei province. At the present time the burial gifts, including the bells, are to be found in the Provincial Museum of Hubei. Reference is often made to the carillon of the Marquis Yi, but this is incorrect as the series of sixty-five comprises one large chiming bell and nine carillons comprising 6, 6, 7, 11, 12, 10, 3, 5 and 4 bells respectively (Figure 7). The largest bell has a height of 1.52 m at a weight of 204 kg; the smallest bell measures 20 cm and weighs 2½ kg. Although Chinese researchers highly commend the purity of tone of these instruments, objective study has shown that this is somewhat exaggerated<sup>5</sup>. This is also apparent in the sui/ku interval.

Figure 8 shows in musical notation both tones for six instruments, including the chiming bell. The staves do not have the function of indicating note values but of linking the sui and ku of each bell to one another. One may then ask oneself the question how many sui/ku intervals occur in classes of 30 cents, starting with 205-235 to 475-505 cents inclusive. The histogram in Figure 9 shows the result for the sixty-four bells. One can clearly see that there is a strong preference for the minor third but also that intervals to and beyond the major third frequently occur. The result is therefore that the average interval of 341 cents is practically halfway between the minor and major third, whereas an average error of a quarter tone is by no means small. In short, from a musical viewpoint the sui/ku interval could definitely have been better. The following question to be asked is what is the relationship between the sui/ku interval and the eccentricity of each bell. In this case we define the eccentricity as the ratio between the length of the lowest long and lowest short axis; therefore this eccentricity is neither linear nor numerical. One can see the result in Figure 10. This shows in no uncertain terms that a relation between the sui/ku interval and the eccentricity can scarcely be identified. There is only a very weak correlation. If one nevertheless wishes to draw a straight line through this cloud of points then this appears to only have a very slight slope that again proves that there is scarcely any question of correlation. The same can be established in other series of bells. Figure 11 shows for example the graph of a carillon of thirteen bells<sup>6</sup>. The scattering is not as large as in the preceding graph but what is more questionable is that the straight line with a slight slope indicates that the sui/ku interval decreases as the eccentricity increases. And that is obviously impossible. In short, what determines that interval between both these hum notes?

---

<sup>4</sup> The grave with all objects found in it is described in the Chinese book *Zeng Hou Yi-mu*, with the English subtitle *Tomb of Marquis Yi of State Zeng*. Two parts (Beijing 1989). Thanks to the numerous photographs and the numerical tables the book is nonetheless reasonably accessible to a person who does not read Chinese. An English summary is given on p. 684-687.

<sup>5</sup> André Lehr, *The tuning of the Bells of Marquis Yi*. In: *Acustica*, Vol. 67, 1988, p.144-148.

<sup>6</sup> The information has been taken from, among other things, the lecture of *Ancient Chinese Two-pitch Bronze Bells* by Ma Chengyan at a *Symposium on The Bronze Age of China* in the *Metropolitan Museum of Art* in New York on 2 and 3 June 1980. See also Wang Shi-xiang in the magazine *Wenwu cankao ziliao*, 1958, no. 1; André Lehr, *Een Chinees klokkenspel uit de 6de eeuw voor Christus*. In: *Klok en Klepel*, no. 14, October 1972, p.6-15.

### **Which parameter determines the sui/ku interval?**

In order to be able to solve our problem, a computer program has been called in to our assistance and namely the ALGOR program with which the frequencies of vibrating objects, therefore of a bell too, can be calculated by means of the so-called finite element method. This is not the place to give a detailed description of the finite elements method<sup>7</sup>. Moreover this is not really necessary, as it is a tool of which only the results count for our study. Nonetheless we would mention that in the present case the Chinese bell is divided into numerous elements (Figure 12-13). These right-angled elements of the thickness of the bell wall considerably facilitate the calculations, albeit that the number of necessary calculations increases substantially. However, this does not pose a problem, as the computer knows how to handle this. For that matter there are several infinite elements programs available. We chose the previously mentioned ALGOR. We calculated by means of this program a series of bells with a cross section in the shape of an ellipse and in the shape of two segments of a circle, and the latter also received special treatment. But more about this later.

The bell in Figure 12 served as basic bell; the view from above is given in Figure 13. In both drawings one sees the elements drawn that are necessary for the calculations. The bell has the fully traditional form, albeit that the underside of the bell is straight and has not been cut out in the shape of a circular. However this is acceptable as this type of bell was also found frequently in ancient China and just as the other type has hum notes which are split up. It should also be noted that the suspension eye has been removed.

The axes at the base of the bell measure 140x180 mm; those in the head measure 80x120 mm. The waist thickness is 5 mm and the thickness of the head is 3 mm. Cast in bell bronze of 80% copper and 20% tin the bell weighs 2.6 kg. Its hum notes are  $f_{is}^2+34$  cents (sui) and  $g_{is}^2+28$  cents (ku). The interval between both tones is therefore 194 cents. This data is also to be found, except for the weight, in table 1 and namely as the first under the group with a cross section of two segments of a circle with sharpened sides.

### **The cross section formed from two segments of a circle or in the shape of an ellipse**

In Figure 13 the horizontal cross section of a traditional Chinese bell is shown and namely in accordance with the specification of the previously mentioned basic bell. One will however notice that both arcs are divided into eighteen equal chords. This division, although not necessary in this amount, is necessary for the calculation. It indicates in fact that the horizontal circumference of the bell is divided into thirty-six elements.

The aim was to subsequently calculate the sui/ku interval for two bells with the same eccentricity, but of which the one has a cross section of an ellipse and the other in the shape of two segments of a circle. The eccentricity is then increased by gradually extending the longest axis. The effect of this is on the one hand an increasingly flat ellipse and on the other hand a bell with increasingly pointed sides. With respect to the latter, Figure 14 shows how this is done. Two chords lying opposite to one another are extended until they cut one another. At that spot the new side of the bell arose. In Figure 14 that is the fourth chord from the side. The axis thus obtained was therefore used for the elliptical cross section. It goes without saying that the other geometrical characteristics of the bell remained unchanged. This is shown in Table 1.

---

<sup>7</sup> For the application of the finite elements to bells see: A.J.G. Schoofs, *Experimental design and structural optimization* (thesis University of Technology Eindhoven, 1987); André Lehr, *Campanologie* (Mechelen, 2nd edition, 1997), p.233-282.

General data					
height (mm)	180				
long axis (mm)	140.0	145.0	155.0	172.6	203.8
corner flank base (deg.)	86.8				
short axis (mm)	100				
thickness flank (mm)	5				
thickness ceiling (mm)	3				
Cross section: an exact ellipse					
linear eccentricity	1.400	1.450	1.550	1.726	2.038
numerical eccentricity	0.699	0.724	0.765	0.815	0.871
sui (cents)	$fis^2+15.9$	$f^2+33.8$	$e^2-43.1$	$cis^2-35.7$	$gis^1-7.4$
ku (cents)	$g^2-23.9$	$fis^2+2.5$	$f^2-46.4$	$d^2+9.3$	$ais^1+27.9$
difference interval (cents)	60.2	68.7	96.7	145.0	235.3
Cross section: two segments of circle with sharpened sides placed against one another					
side corner (degrees)	132.9	112.5	97.1	78.4	64.2
sui (cents)	$fis^2+34.0$	$f^2+18.0$	$dis^2-16.9$	$b^1+26.0$	$g^1-20.6$
ku (cents)	$gis^2+28.3$	$gis^2+22.6$	$gis^2-19.3$	$fis^2+44.8$	$dis^2+42.9$
difference interval (cents)	194.3	304.6	497.6	718.8	863.5

**Table 1**

Five bells according to this principle are mentioned in Table 1. The first is the standard bell, the next bell has a side that arose by extending both following chords, in this case therefore the second chord from the side. The last bell was therefore obtained by extending the fifth chord. This table also shows how as a result of this the side corner of almost  $133^\circ$  slowly decreases to more than  $64^\circ$ . The effect on the separation of the hum note appears to be exceptionally large, and certainly in comparison to the elliptical cross section. This is illustrated in Figure 15. One clearly sees how in the case of increasing eccentricity both hum notes for the cross section of two segments of a circle separate much quicker than in the case of an ellipse. Evidently it is not in the first instance the eccentricity of the cross-section that is determinative for the sui/ku interval but the pointedness of the side of the bell.

Figure 16 confirms this in a different way. In this diagram the hum note of the standard bell has been set at 0 cents for both the elliptical cross section ( $fis^2+15.9$ ) and the cross section with segments of a circle ( $fis^2+34.0$ ).

All measurements are calculated according to this basis. The conclusion is subsequently clear. Both bell types have each time the same eccentricity, so that evidently the corner of the side is determinative for the extent to which both hum notes separate. Or vice versa, the rounder the side the nearer both hum notes are to one another. Apparently this affects hum notes as in the case of the elliptical cross section both hum notes are lowered; in the case of segments of a circle too, but to a much lesser degree. The conclusion is therefore, the more pointed the side corner, therefore the more local the alteration of the bell form, the further the sui and ku will be separated from one another. Or more generally, if one wishes to separate both hum notes over a significant interval, then a strictly local alteration must be placed in the circumference. In all studies on this subject it is precisely this essential element of the Chinese bell that has been overlooked! The correctness of this proposition will for that matter be verified in another way in the consideration of the tuning of the bell.

### **The two pitches in a historical perspective**

It is certain that the Chinese musicians at that time made a conscious musical use of the two-pitch nature of their bells. This is evidenced by for example inscriptions on some bells<sup>8</sup>. But did they also know how to effect the required splitting up?

If one examines the bell cross diameters, then the pointed side often appears to not completely conform to the form of a segment of a circle. The sides are sometimes somewhat sharpened or precisely the opposite, somewhat more blunt than the exact form allows. There is no evidence to show that such has been effected with a view to the interval between hum notes nor can it be made credible by reason of the complex phenomenon. One must rather assume that the alteration of the side corner happened during the moulding process in order to be able to mould more easily. It is only a hypothesis as the scarce sources on bell designing, although these sources even date back to the Chou dynasty<sup>9</sup>, only mention the profile, in this case the vertical cross section, and not the form of the horizontal cross section. It may be that this part received little attention. We do know that casters sometimes even tuned their own cast bells, i.e. by altering the model to as yet be able to realise the correct sui and the correct ku. However, altering the side corner did not effect this but by locally thinning the wall and for that reason, this is the subject of the following paragraph.

### **The tuning of a Chinese bell**

Although research shows that the majority of the Chinese bells have a non-variable wall thickness, there is a not inconsiderable amount of bells in which this is precisely the opposite, in particular bells that were used in a carillon. These bells were tuned. Figures 17-18 show that this was done by locally thinning the wall and namely by means of vertical grooves as narrow bands on the inside of the bell. The position of these grooves is clearly related to the position of the antinodes of the sui or ku. The effect thereof is obvious. One understands this by analogy with the tuning of a tuning rod of a xylophone or metallophone. One can lower the tone of such a rod by thinning it in the middle, precisely in the antinode of hum note of that rod. For that reason too a thinning in the antinode of the sui will lower this tone, whereas a thinning in the antinode of the ku will lower that tone. But there are also overlaps. Because when one thins the wall halfway in a sui antinode or a ku antinode, one may expect that both

---

<sup>8</sup> Lothar von Falkenhausen, *Suspended Music. Chime-Bells in the Culture of Bronze Age China* (Berkeley, Los Angeles, Oxford, 1993), p.280-309.

<sup>9</sup> Le Tcheou-Li ou Rites des Tcheou. Traduit pour la première fois du Chinois par feu Edouard Biot, Tome 2 (Paris, 1851), p.498-503.

tones will be lowered, albeit to a lesser extent. Sometimes this may be the desired effect but then again sometimes it may not.

It seemed worthwhile to re-check this by means of computer calculations. And we decided for that matter to choose a small groove instead of a wide groove on which bronze is removed. This vertical groove was placed on the inside from the bottom of the bell. The height was 100 mm for a three-cornered cross section, of which the base was 20 mm and the depth 2½ mm. The pointed side of the groove extends therefore to one half of the wall thickness. The weight thus removed is approximately 20 g, therefore almost 1% of the bell weight of our standard bell.

	<b>original</b>	<b>groove halfway</b>	<b>groove in waist</b>	<b>groove in side</b>
groove in the antinode of		ku	sui	sui
sui (cents)	$f^{is^2}+34.0$	$f^2+34.6$	$f^2-18.4$	$d^2+44.9$
ku (cents)	$gis^2+28.3$	$g^2-25.7$	$gis^2+30.9$	$gis^2+20.4$
difference interval (cents)	194.3	139.7	349.3	575.5

**Table 2**

Table 2 makes it clear that both hum notes are lowered, although not to an equal degree. The hum note that is lowered the most has the groove in an antinode. Moreover, it appears that the groove between the front and side, i.e. in the antinode of the ku, has less effect on the high hum note than a groove in the front and side, i.e. in the antinode of the sui, has on the low component. But it should be clear that this was a good means with which to control the absolute pitches of both hum notes and the interval between the two. Nonetheless, the results were poor. On the basis of the wide tuning grooves it would seem obvious to assume that one failed to sufficiently understand the effect of the width of the tuning grooves in particular. On the other hand the impurities do not need to have been caused by the tuning method but may also have been due to the fact that the casters and musicians at that time did not have good measuring instruments. Moreover, one should realise that the Chinese bell is struck in such a way that only the hum note really develops. For that reason, the risk of dissonance of two simultaneously sounding, mutually false bells, is scarcely to be feared. It is a different story with organ pipes and strings where the entire series of overtones reinforces the hum note. Moreover, the overtones of these instruments stand in a fixed relationship to their hum note, so that when the hum note for example is too low, all over tones will be too low. And that will certainly have implications.

And finally, the bells were by no means always tuned. In many cases one put up with the deviations, particularly in the case of burial gifts.

### **The dewarbling of Western bells**

The Chinese bell casters filed vertical grooves in their bells in order to influence the sui/ku interval, for example to make this smaller. It is therefore possible in principle to have the sui and ku coincide, assuming at least that one has not already gone through the bell wall. But it is clear that this method of working surprisingly corresponds to the so-called *dewarbling* of European bells, a method that was developed by the author in 1953<sup>10</sup>. This concerns the following matter.

Bells often have a beating sound. This can be striking, particularly in the hum note as we then hear that the hum note sharply fluctuates in pitch and volume, for example several times per second. This is caused by the fact that the symmetry has been lost due to a local blow hole in the bell, and in consequence the hum note and the overtones divide into two components. As a blow hole practically always introduces a slight asymmetry, both hum notes will be so near to one another that they cannot be listened to separately. But their presence does manifest itself in a warble, of which the number corresponds per seconds to the frequency difference between both components. It is by now clear that the symmetry in the Chinese bell is also displaced but to a much greater degree so that we can nonetheless separately listen to both components, sui and ku, even if they sound simultaneously.

The warbling itself is really not difficult. One makes a vertical groove in the antinode of the highest hum note and with it simultaneously in the node of the lowest hum note. Thus the highest hum note will be lowered and the lowest will remain unchanged. It should be clear that through a suitable choice of the length and depth of the vertical groove, both hum notes will finally coincide. Thus the bell has become free of warbles.

The discovery of this method occurred independently from the Chinese method of working because not only was at that time in the West very little known about the acoustic of the Chinese bells, but even the Chinese researchers did not know how their distant ancestors had tuned bells. Therefore, the discoveries that the Chinese had made in ancient times were independently re-discovered three thousand years later in the Netherlands. The Chinese used this method to tune their bells, in the Netherlands on the other hand it was used to correct a beating bell sound.

However, one must add to this that the Chinese had, judging by the tuning marks in their bells (Figure 17-18) and the large scattering in the sui/ku interval (Figure 10), evidently failed to understand that the groove has to be as narrow as possible if one wishes to leave the other component unchanged. In the case of dewarbling on the other hand, this was soon clear. After all, if a too wide vertical groove is chosen, the lowest hum note will also be affected. And that is not intended.

### **A new type of bell?**

The phenomenon used in the Chinese antiquity that an asymmetrical bell has two tones could perhaps also be of value for the European bell, and for the carillon in particular. After all one could suffice with half the bells or use the second tone as added register. Such a bell, as we now know, must have a very local asymmetrical structure, at least if one wishes to place both bell tones at an adequate interval, for example at a fifth from one another. A bell with a right-angled or elliptical cross section will never be able to conform to this, but a bell with for example four vertical fins placed at four places in the circumference probably will. But at the same time we are faced with another problem. The hum note has four nodes and antinodes in the circumference. However in the carillon the overtones play almost as important a role as

---

<sup>10</sup> André Lehr, *Campanologie* (Mechelen, 2nd edition, 1997), p.167-173.

the hum note. But these tones vibrate in sixes, eighths, tens etc. Moreover, they must have a harmonious musical relationship with one another ( $c^1$ - $c^2$ - $es^2$ - $g^2$ - $c^3$  etc. if  $c^1$  is the hum note).

Therefore, it is anything but an easy task to design on the basis of this a two-pitch bell with a strong local disturbance in the symmetry.

### **Captions of the figures**

1. A traditional Chinese carillon at the time of the Chou dynasty (c.1030-221 BC) from the collection of the National Carillon Museum.
2. Wooden hammer to play a carillon from the tomb of Marquis Yi (433 BC or shortly afterwards). (*Zeng Hou Yi-mu, Tomb of Marquis Yi of State Zeng*. Part 1, Beijing 1989, p. 133).
3. Way in which a carillon is played. (Lothar von Falkhausen, *Suspended Music. Chime-Bells in the Culture of Bronze Age China*. Berkeley, Los Angeles, Oxford 1993, p.30).
4. Way in which a carillon is played. Lothar von Falkhausen, *Suspended Music. Chime-Bells in the Culture of Bronze Age China*. Berkeley, Los Angeles, Oxford 1993, p.212.
5. Several views of a Chinese bell. (*Zeng Hou Yi-mu, Tomb of Marquis Yi of State Zeng*. Part 1, Beijing 1989, p.90).
6. The horizontal cross section of an antique Chinese bell. The figure on the left shows the vibration mode of the sui. The figure on the right shows the vibration mode of the ku.
7. A drawing of the original arrangement of the carillon of the Marquis Yi (433 BC or shortly afterwards). (*Zeng Hou Yi-mu, Tomb of Marquis Yi of State Zeng*. Part 1, Beijing 1989, p. 76).
8. The sui and ku of a number of carillons from the grave of Marquis Yi (433 BC or shortly afterwards). A horizontal staff links the staffs of the sui and ku that belong together to one another. (*Zeng Hou Yi-mu, Tomb of Marquis Yi of State Zeng*. Part 1, Beijing, 1989, p.76).
9. A histogram of the sui/ku interval of the sixty-four bells of Marquis Yi in classes of thirty cents.
10. The relation between the sui/ku interval and the eccentricity of the sixty-four bells of Marquis Yi.
11. The relation between the sui/ku interval and the eccentricity of a carillon comprising thirteen bells. Information taken from Wang Shi-xiang in *Wenwu cankao ziliao*, 1958, no.1.
12. The model bell that is used in the calculations with the Finite Elements Method (ALGOR). For that purpose the bell has been divided into a large number of components, called elements.
13. The view from above of the model bell in Figure 12 also divided into elements.
14. The method to make the sides increasingly pointed. It is shown how the four chords on the fourth place from the side determine the new long diameter (bell 4 in table 1).
15. The interval in cents between the sui and ku at a cross section according to an ellipse (lowest line) and two segments of a circle placed against one another (uppermost line).
16. The interval in cents of the sui and ku in each bell in relation to the sui of respectively the bell with an elliptical cross section (drawn line) and with a cross section of two segments of a circle (marked line).
17. The places on the inside where the Chinese bell was usually tuned. According to *Zeng Hou Yi-mu (Tomb of Marquis Yi of State Zeng)*. Part 1 (Beijing 1989), p.91.
18. The places on the inside where the Chinese bell was usually tuned. According to *Zeng Hou Yi-mu (Tomb of Marquis Yi of State Zeng)*. Part 1 (Beijing 1989), p.98.

